

Neuro-Symbolic Complex Event Recognition: Part I

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<https://cer.iit.demokritos.gr>



About the Tutorial

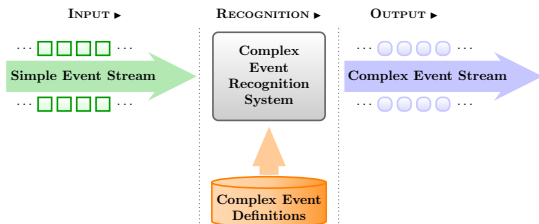
Structure:

- ▶ Part I: Symbolic AI for complex event recognition.
- ▶ Part II: Integration of symbolic with sub-symbolic AI for complex event recognition.

Slides, code, data & opportunities for collaboration:

<https://cer.iit.demokritos.gr>

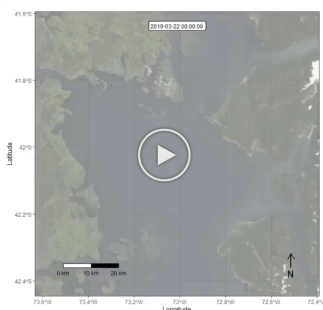
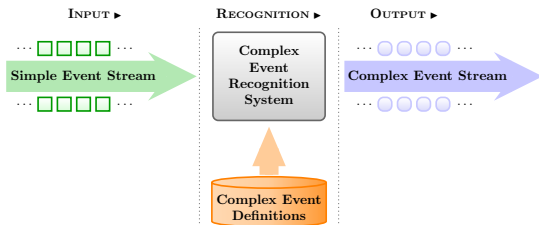
Complex Event Recognition (Event Pattern Matching)^{*,†}



^{*} Giatrakos et al, Complex Event Recognition in the Big Data Era: A Survey. VLDB Journal, 2020.

[†] Alevizos et al, Probabilistic Complex Event Recognition: A Survey. ACM Computing Surveys, 2017.

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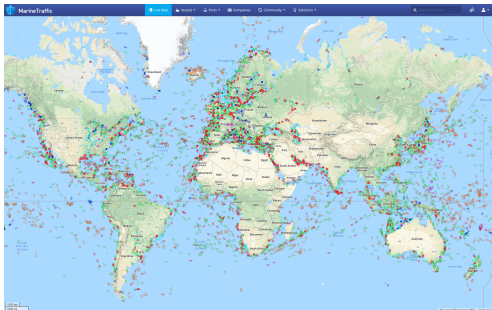


<https://rdcu.be/cNkQE>

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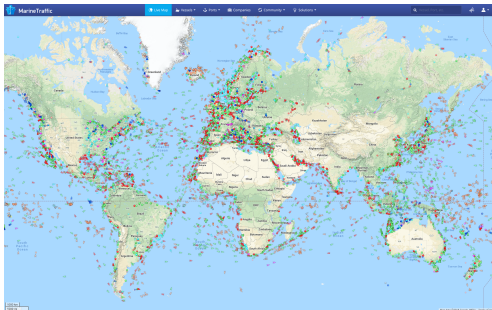
Maritime Situational Awareness*



<http://www.marinetraffic.com>

* Artikis and Zissis, Guide to Maritime Informatics, Springer, 2021.

Maritime Situational Awareness*



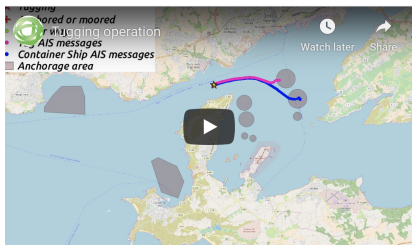
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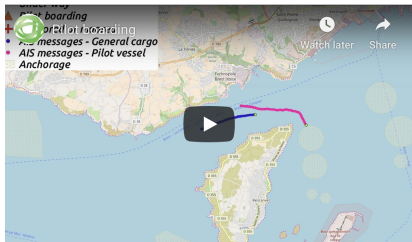
<https://cer.iit.demokritos.gr> (fishing vessel)

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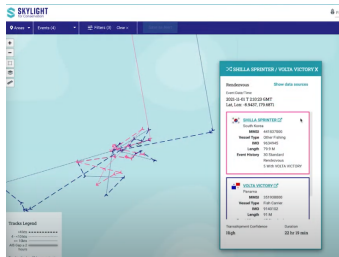
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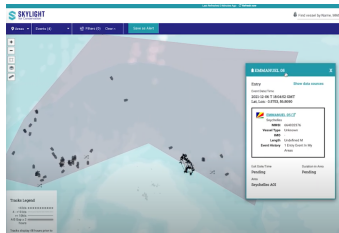
<https://cer.iit.demokritos.gr> (tugging)



<https://cer.iit.demokritos.gr> (pilot boarding)



<https://www.skylight.global> (rendez-vous)



<https://www.skylight.global> (enter area)

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Data Challenges

- ▶ **Velocity, Volume:** Millions of position signals/day at European scale.

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- ▶ Lack of **Veracity:** GPS manipulation, vessels reporting false identity, communication gaps.
- ▶ **Distribution:** Vessels operating across the globe.

Many Other Applications

- ▶ Cardiac arrhythmia recognition.
- ▶ Financial fraud detection.
- ▶ Human activity recognition.
- ▶ Intrusion detection in computer networks.
- ▶ Traffic congestion recognition and forecasting in smart cities.

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- ▶ Reasoning under uncertainty
 - ▶ to deal with various types of noise.
- ▶ Complex event forecasting
 - ▶ to support proactive decision-making.

Complex Event Recognition vs Database Management Systems*

Complex event recognition systems:

- ▶ Process data without storing them.

*Gugola and Margara, Processing Flows of Information: From Data Stream to Complex Event Processing. ACM Computing Surveys, 2012.

Complex Event Recognition vs Database Management Systems*

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 - ▶ Queries deployed once and executed continuously until removed.
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- ▶ Latency requirements are very strict.

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We have [Deep Learning](#) and it seems to work. Can we go home?

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- ▶ Explanation — why did we detect a complex event?
- ▶ **Machine Learning** is necessary. But:
 - ▶ Complex events are rare.
 - ▶ Supervision is scarce.
- ▶ More often than not, background knowledge is available — let's use it!

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Event Calculus*

- ▶ A **logic programming language** for representing and reasoning about events and their effects.
- ▶ Key components:
 - ▶ **event** (typically instantaneous).
 - ▶ **fluent**: a property that may have different values at different points in time.

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 - ▶ **fluent**: a property that may have different values at different points in time.
- ▶ Built-in representation of **inertia**:
 - ▶ $F = V$ holds at a particular time-point if $F = V$ has been *initiated* by an event at some earlier time-point, and not *terminated* by another event in the meantime.

*Kowalski and Sergot, A Logic-based Calculus of Events. New Generation Computing, 1986.

Run-Time Event Calculus (RTEC)*

initiatedAt($F = V, T$) \leftarrow
happensAt(E_{In_1}, T),
[conditions]

...

initiatedAt($F = V, T$) \leftarrow
happensAt(E_{In_i}, T),
[conditions]

terminatedAt($F = V, T$) \leftarrow
happensAt(E_{T_1}, T),
[conditions]

...

terminatedAt($F = V, T$) \leftarrow
happensAt(E_{T_j}, T),
[conditions]

where

conditions: $0-K$ **happensAt**(E_k, T),
 $0-M$ **holdsAt**($F_m = V_m, T$),
 $0-N$ atemporal-constraint _{n}

* Artikis et al, An Event Calculus for Event Recognition. IEEE TKDE, 2015.
<https://github.com/aartikis/RTEC>

Run-Time Event Calculus (RTEC)

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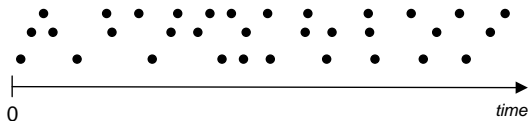
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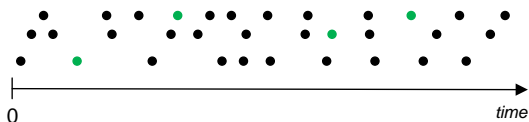
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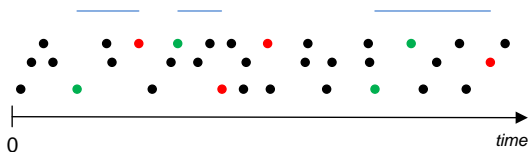
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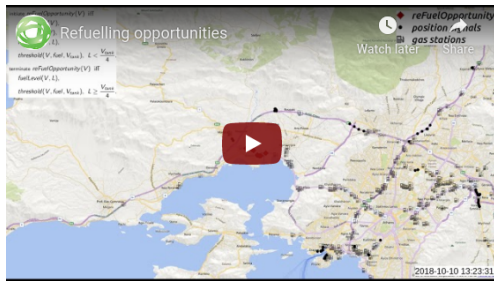
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terminatedAt($F = V, T$) \leftarrow
happensAt(E_{T_j}, T),
[conditions]

holdsFor($F = V, I$)



Fleet Management*



<https://cer.iit.demokritos.gr> (refuelling opportunities)

*Tsilionis et al, Online Event Recognition from Moving Vehicles. Theory and Practice of Logic Programming, 2019.

RTEC: Interval-based Reasoning

holdsFor(*anchoredOrMoored*(*Vessel*) = true, *I*) \leftarrow
 holdsFor(*stopped*(*Vessel*) = *farFromPorts*, *I_{sf}*),
 holdsFor(*withinArea*(*Vessel*, *anchorage*) = true, *I_{wa}*),
 intersect_all([*I_{sf}*, *I_{wa}*], *I_{sa}*),
 holdsFor(*stopped*(*Vessel*) = *nearPorts*, *I_{sn}*),
 union_all([*I_{sa}*, *I_{sn}*], *I*).

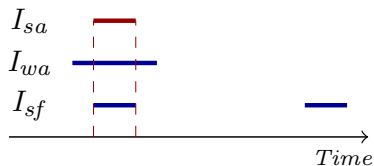
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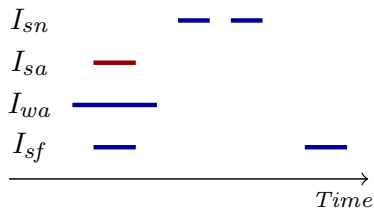
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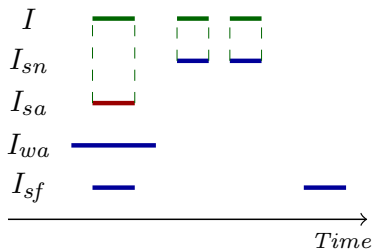
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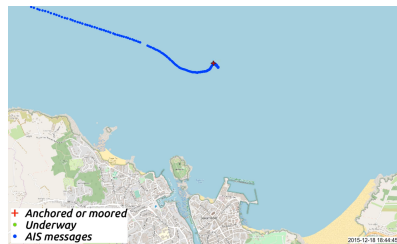
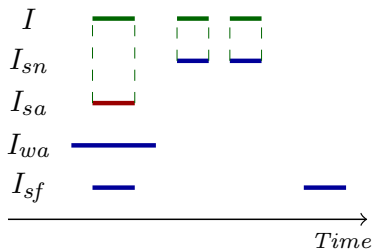
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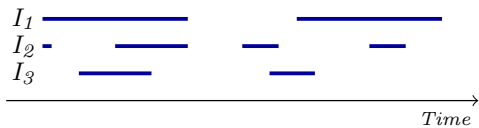
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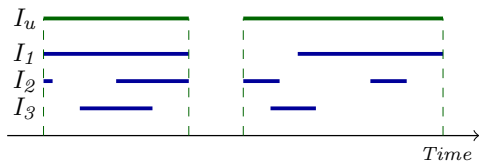
<https://cer.iit.demokritos.gr> (anchored or moored)

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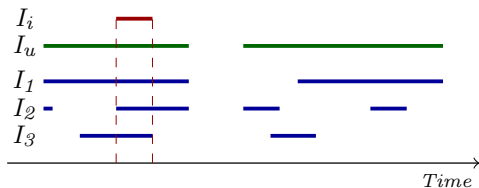
union_all([I_1, I_2, I_3], I_u)



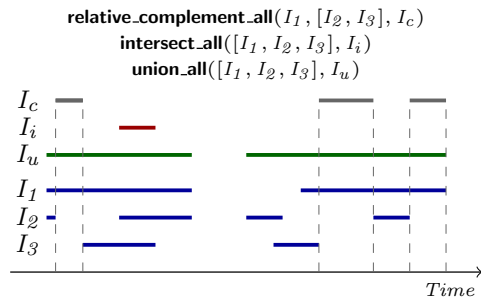
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intersect_all($[I_1, I_2, I_3], I_i$)

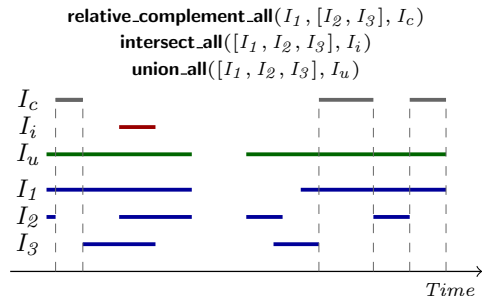
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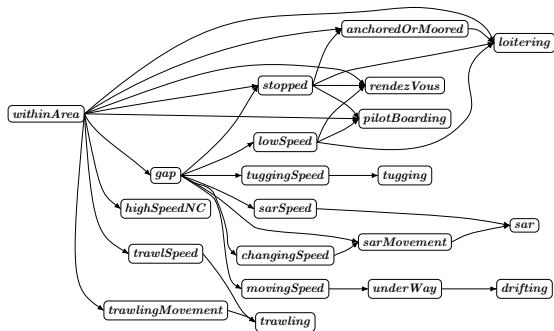
RTEC: Interval-based Reasoning & Allen Relations*



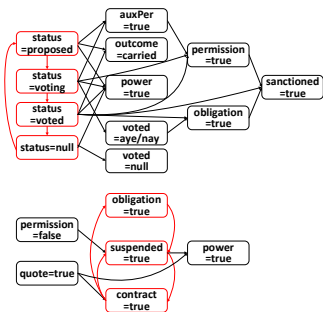
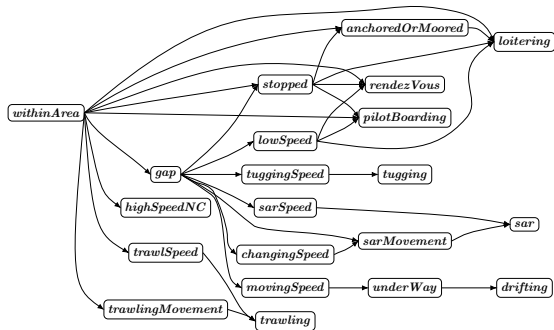
Relation	Illustration
$\text{before}(i^s, i^t)$	
$\text{meets}(i^s, i^t)$	
$\text{starts}(i^s, i^t)$	
$\text{finishes}(i^s, i^t)$	
$\text{during}(i^s, i^t)$	
$\text{overlaps}(i^s, i^t)$	
$\text{equal}(i^s, i^t)$	

* Mantenoglou et al, Complex Event Recognition with Allen Relations. Knowledge Representation and Reasoning (KR), 2023.

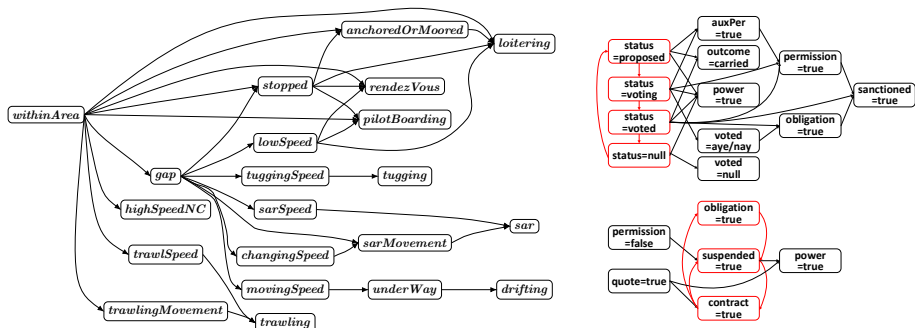
Semantics



Semantics



Semantics

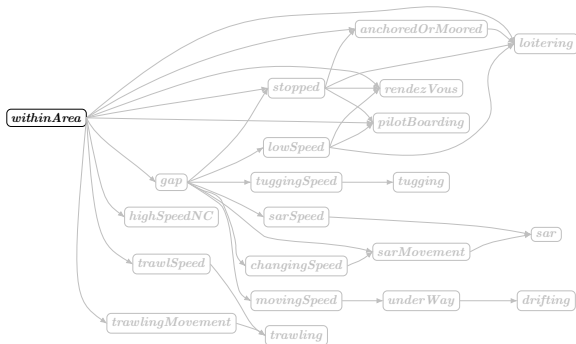


Proposition

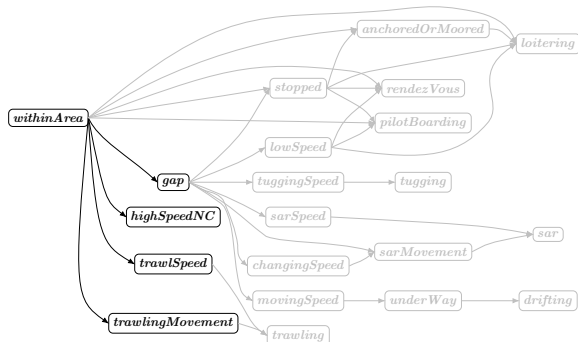
An event description in RTEC is a locally stratified logic program*.

*Mantenoglou et al, Stream Reasoning with Cycles. Knowledge Representation and Reasoning (KR), 2022.

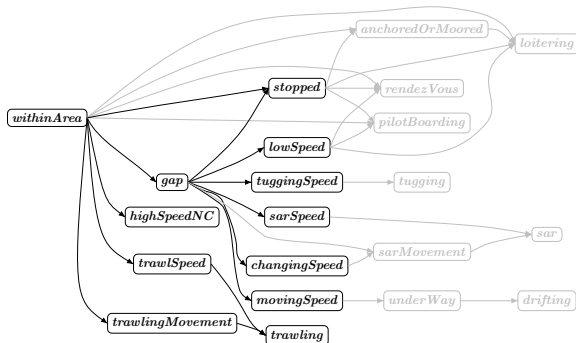
Stratification & Reasoning



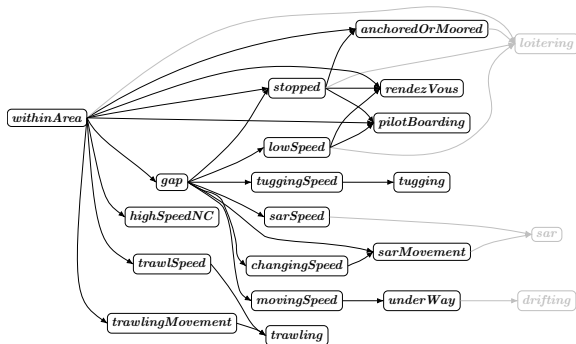
Stratification & Reasoning



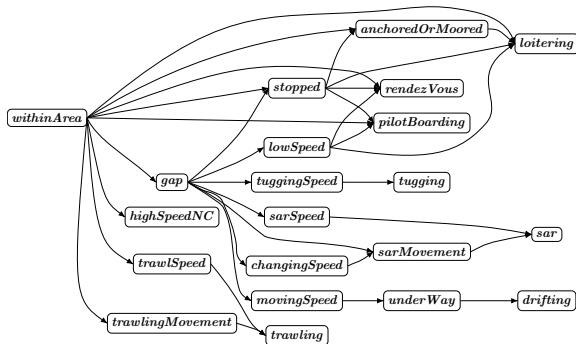
Stratification & Reasoning



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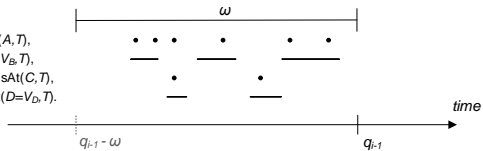


Windowing

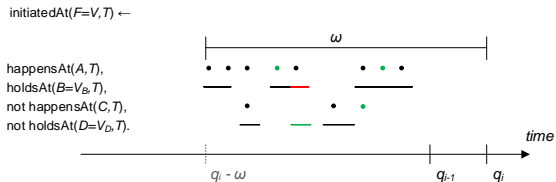
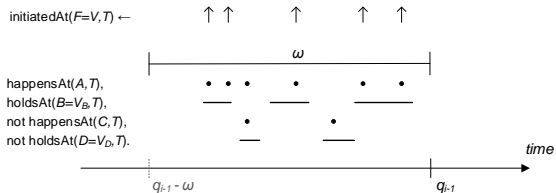
initiatedAt($F=V, T$) ←

↑ ↑ ↑ ↑ ↑

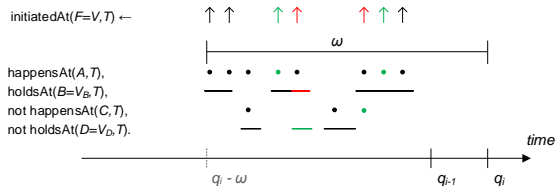
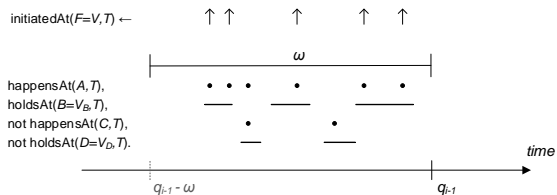
happensAt(A, T),
holdsAt($B=V_B, T$),
not happensAt(C, T),
not holdsAt($D=V_D, T$).



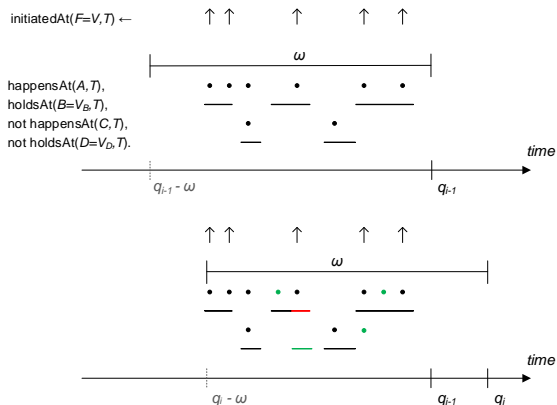
Windowing



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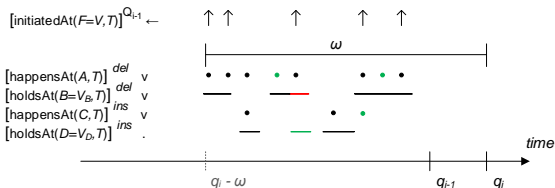
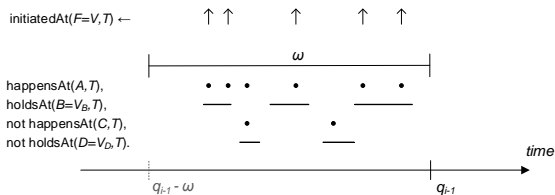


Incremental Reasoning: Deletion Phase*



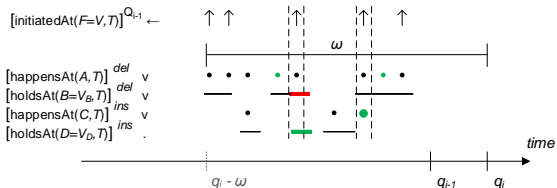
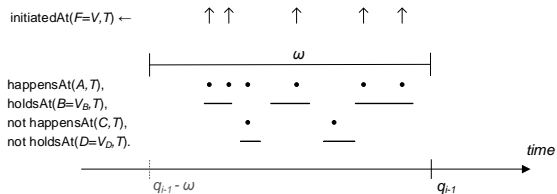
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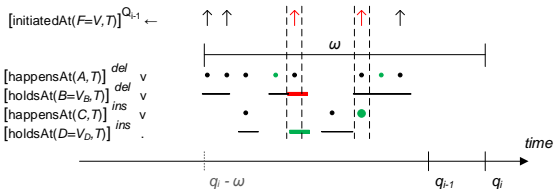
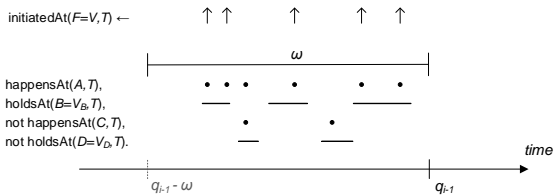
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RTEC: Correctness and Complexity

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The time to compute the maximal intervals of a fluent is linear to the window size.

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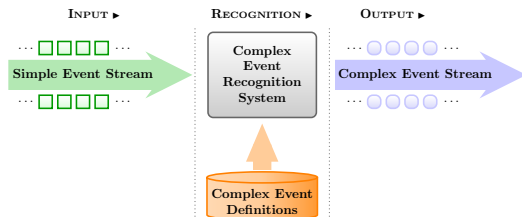
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- ▶ Direct routes to probabilistic reasoning → handle the lack of veracity of data streams.

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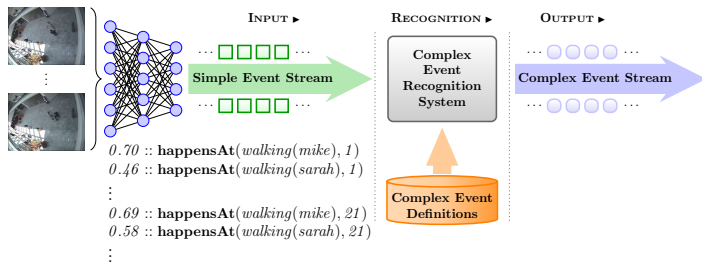
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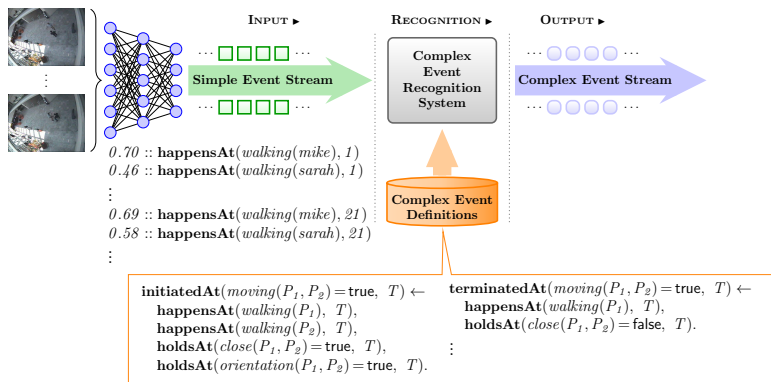
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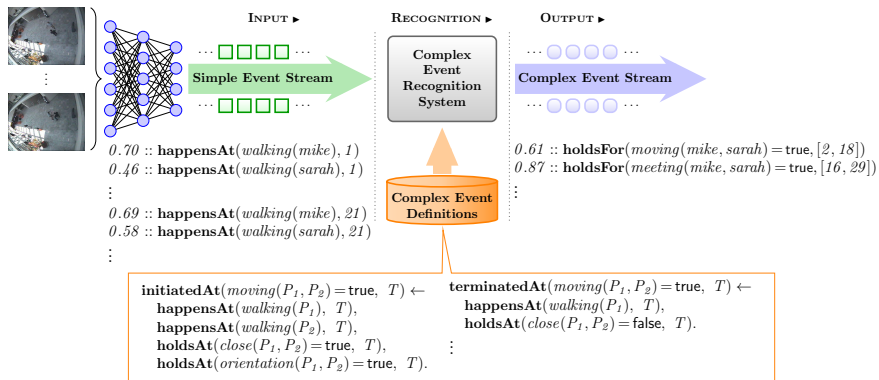
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